Math 112 – Review for Test 3

Directions: Show your work for full credit. The way you drive your answer is most important.

1. Solve the following differential equations:
   (a) $\frac{dy}{dt} = y^2 \sin(t)$ satisfying $y(0) = 1/2$.
   (b) $y' = yt$ satisfying $y(0) = 1$.
   (c) $t\frac{dx}{dt} = (1 + 2\ln t)\tan x$. (Assume $x, t > 0$.)
   (d) $x(x+1)\frac{du}{dx} = u^2$ satisfying $u(1) = 1$.
   (e) $\frac{dy}{dt} = -y\ln(\frac{y}{2})$ satisfying $y(0) = 1$.

2. In psychology, the Weber-Fechner Law describes a person's response to a stimulus as follows: the rate of change of the reaction R with respect to the stimulus S is inversely proportional to the stimulus. Solve the differential equation and examine the behaviour of the response curve.

3. A cubic ice cube with side length a melts at a rate proportional to its surface area.
   (a) Write a differential equation of its volume, V.
   (b) If the initial volume is V_0, solve the differential equation and graph the solution.
   (c) When does the ice cube disappear?

4. Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.
   (a) If $P(0) = 500$, find $P(t)$ in terms of k and t.
   (b) If $P(2) = 700$, find k.
   (c) Find the value that $P(t)$ approaches as t gets very large.

5. (a) Find the 6th degree McLaurin polynomial for $f(x) = x\sin x$.
   (b) Use Taylor's theorem to bound the error in approximating the function $f(x) = x\sin(x)$ on the interval $[0, \pi/2]$.
   (c) Find the actual largest error committed by $M_6(x)$ on the same interval.

6. Using the fact that $d(\arctan x)dx = 1/(1+x^2)$ and $\arctan 1 = \pi/4$, approximate the value of π using the third-degree Taylor polynomial of $4\arctan x$ about $x = 0$.

7. Determine if the following integral are convergent or divergent, If they converge, evaluate them.
   (a) $\int_1^\infty \frac{dx}{x\sqrt{1+x^2}}$
   (b) $\int_5^\infty \frac{dx}{(x-1)^{3/2}}$
   (c) $\int_0^\infty \frac{x^2 dx}{4x^3+5}$
   (d) $\int_0^6 \frac{dx}{(x-4)^{2/3}}$

8. Use convergence test to determine if the following integrals converge or diverge.
   (a) $\int_1^\infty \frac{dx}{x^3+1}$
   (b) $\int_0^1 \frac{dx}{x^{19/20}}$
   (c) $\int_1^\infty \frac{du}{u+u^2}$
   (d) $\int_0^\infty \frac{dy}{1+e^y}$

(e) $\int_0^\infty \frac{e^{-x}dx}{\sqrt{x}}$

(f) $\int_1^\infty \frac{4dx}{x(x+1)}$

(g) $\int_1^\infty \frac{dx}{\sqrt{x+x^3}}$

(h) $\int_2^\infty \frac{dx}{x(\ln x)^p}$ for $p > 1$

(i) $\int_2^\infty \frac{dx}{x(\ln x)^p}$ for $p < 1$

(j) $\int_2^\infty \frac{dx}{x(\ln x)^p}$ for $p = 1$

(k) $\int_1^2 \frac{dx}{x(\ln x)^p}$ for $p > 1$

(l) $\int_1^2 \frac{dx}{x(\ln x)^p}$ for $p < 1$

(m) $\int_1^2 \frac{dx}{x(\ln x)^p}$ for $p = 1$

9. Compute the following limits:

(a) $\lim_{x\to 0^+} x \ln x$

(b) $\lim_{x\to\infty} \frac{1+2x}{\sqrt{x}}$

(c) $\lim_{x\to 0^+} \frac{3^{\sin x}-1}{x}$

(d) $\lim_{x\to\infty} e^{-x}x^{1/2}$

(e) $\lim_{x\to\infty}(\ln x)^{1/x}$

(f) $\lim_{x\to 1^+} x^{\frac{1}{1-x}}$

(g) $\lim_{x\to\infty}(\ln x)^{1/x}$

10. Do the following sequences converge or diverge, if they do find their limit.

(a) $a_n = \{\frac{3n}{2n-1}\}_{n=1}^\infty$

(b) $a_n = \{\sqrt{\frac{7n}{n-4}}\}_{n=5}^\infty$

(c) $a_n = \{\frac{(2n+5)^2}{n^2}\}_{n=1}^\infty$

(d) $a_n = \{\frac{n^3}{e^n}\}_{n=1}^\infty$

(e) $a_n = \{\frac{n}{10} + \frac{10}{n}\}_{n=1}^\infty$

(f) $a_n = \{\frac{(-1)^n}{n}\}_{n=1}^\infty$

11. You should be comfortable with finding the sums of infinite geometric series like in the following examples:

(a) $\Sigma_{n=1}^\infty \frac{4^n}{5^{n+1}}$

(b) $\Sigma_{n=1}^\infty \left(\frac{1}{3}\right)^n$

(c) $\Sigma_{n=0}^\infty \left(\frac{-1}{2}\right)^n$